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Aufgabe 16.3.6

zz: $\int_{\Omega} \{\varphi(x) + \psi(x)\} d\ell_n(x) = \int_{\Omega} \varphi(x) d\ell_n(x) + \int_{\Omega} \psi(x) d\ell_n(x)$

mit: $\varphi(x) = \sum_{i=1}^s c_i \chi_{\Omega_i}(x)$, $\psi(x) = \sum_{j=1}^t d_j \chi_{\Theta_j}(x)$, $\Omega = \bigcup_{i=1}^s \Omega_i = \bigcup_{j=1}^t \Theta_j$ (disjunkte Zerlegung)

Bew: $\varphi(x) + \psi(x) = \sum_{i=1}^s c_i \sum_{j=1}^t \chi_{\Omega_i \cap \Theta_j}(x) + \sum_{j=1}^t d_j \sum_{i=1}^s \chi_{\Omega_i \cap \Theta_j}(x)$ $\Omega = \bigcup_{i,j} \underbrace{\Omega_i \cap \Theta_j}_{=: \Sigma_{i,j}}$

$$= \sum_{i=1}^s \sum_{j=1}^t (c_i + d_j) \chi_{\Sigma_{i,j}}(x)$$

$$\Rightarrow \int_{\Omega} \varphi(x) + \psi(x) d\ell_n(x) = \int_{\Omega} \sum_{i=1}^s \sum_{j=1}^t (c_i + d_j) \chi_{\Sigma_{i,j}}(x) d\ell_n(x)$$

$$= \sum_{i=1}^s c_i \ell_n^*(\Omega_i) + \sum_{j=1}^t d_j \ell_n^*(\Theta_j)$$

$$= \int_{\Omega} \varphi(x) d\ell_n(x) + \int_{\Omega} \psi(x) d\ell_n(x) \quad \square$$