Kapitel 19.1, Aufgabe 5 (Paul Schöner, Stefanie Schroeder, Julia Hasselwander)

$f_i \in C^1(\mathbb{R}^3, \mathbb{R})$, gesucht sind die Gradienten und deren Auswertung bei $(x_0, y_0, z_0) \in \mathbb{R}^3$

(i) $f_1(x,y,z) = x^3 y - yz^2 + e^{xyz}$, $(x_0, y_0, z_0) = (1,1,1)$

$$\operatorname{grad} f_1(x,y,z) = \left(\frac{\partial f_1}{\partial x}, \frac{\partial f_1}{\partial y}, \frac{\partial f_1}{\partial z}\right) = \left(3x^2 y + yze^{xyz}, \; x^3 - z^2 + xze^{xyz}, \; -2yz + xye^{xyz}\right)$$

$$\Rightarrow \operatorname{grad} f_1(x_0, y_0, z_0) = (3+e, e, -2+e)$$

(ii) $f_2(x,y,z) = \sqrt{1+x^2+y^2} - \ln(1+z^2)$, $(x_0, y_0, z_0) = (0,0,1)$

$$\operatorname{grad} f_2(x,y,z) = \left(\frac{\partial f_2}{\partial x}, \frac{\partial f_2}{\partial y}, \frac{\partial f_2}{\partial z}\right) = \left(2x \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{1+x^2+y^2}}, \; 2y \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{1+x^2+y^2}}, \; -2z \cdot \frac{1}{1+z^2}\right)$$

$$= \left(\frac{x}{\sqrt{1+x^2+y^2}}, \frac{y}{\sqrt{1+x^2+y^2}}, -\frac{2z}{1+z^2}\right)$$

$$\Rightarrow \operatorname{grad} f_2(x_0, y_0, z_0) = \left(\frac{0}{\sqrt{1+0^2+0^2}}, \frac{0}{\sqrt{1+0^2+0^2}}, \frac{-2 \cdot 1}{1+1^2}\right) = (0, 0, -1)$$