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2.2 : Es seien $\Omega \subseteq \mathbb{R}^3$ und $\varphi \in C^2(\Omega, \mathbb{R})$ ein Potential des Gradientenfeldes $f : \Omega \to \mathbb{R}^3$.

Dann : $\operatorname{rot} f(x) = 0$ in Ω. d.h. $f(x) = \nabla \varphi(x)$.

Beweis : $\operatorname{rot}(f(x)) = \operatorname{rot}(\nabla \varphi(x)) = \operatorname{rot}(\partial_x \varphi, \partial_y \varphi, \partial_z \varphi)$

$$= \left(\frac{\partial \varphi_z}{\partial y} - \frac{\partial \varphi_y}{\partial z}, \frac{\partial \varphi_x}{\partial z} - \frac{\partial \varphi_z}{\partial x}, \frac{\partial \varphi_y}{\partial x} - \frac{\partial \varphi_x}{\partial y} \right)^T$$

$$\overset{*}{=} (\varphi_{zy} - \varphi_{yz}, \varphi_{xz} - \varphi_{zx}, \varphi_{yx} - \varphi_{xy})^T$$

$$= (0, 0, 0) = 0 \quad \square.$$

Satz von Schwarz

$\varphi \in C^2(\mathbb{R}^n, \mathbb{R})$,

$\Rightarrow \frac{\partial^2 \varphi}{\partial x_i \partial x_j} = \frac{\partial^2 \varphi}{\partial x_j \partial x_i}$, $i, j = 1, \dots, n$

$\Leftrightarrow \partial_{ij} \varphi = \partial_{ji} \varphi$