22.

$$L[c] = \int_a^b |c'(t)| = \int_a^b \sqrt{g_{11}\,\dot{c}_1(t)^2 + 2g_{12}\,\dot{c}_1(t)\dot{c}_2(t) + g_{22}\,\dot{c}_2(t)^2}\,dt$$

Bew:

$$c(t) = X(\gamma(t)) = X(u(t), v(t))$$

$$\Rightarrow c'(t) = X_u \cdot u'(t) + X_v \cdot v'(t)$$

$$|c'| = \sqrt{|c'|^2} = \sqrt{(X_u \cdot u'(t) + X_v \cdot v'(t))^2}$$

$$= \underbrace{X_u^2}_{g_{11}} \cdot u'(t)^2 + 2\underbrace{X_u \cdot X_v}_{g_{12}} \cdot u'(t) \cdot v'(t) + \underbrace{X_v^2}_{g_{22}} \cdot v'(t)^2$$

$$= \sqrt{g_{11} \cdot u'(t)^2 + 2g_{12} \cdot u'(t) \cdot v(t) + g_{22} \cdot v'(t)^2}$$

$$= \sqrt{g_{11} \cdot \dot{c}_1(t)^2 + 2g_{12} \cdot \dot{c}_1(t) \cdot c_2(t) + g_{22} \cdot \dot{c}_2(t)^2}$$

$\square$

b)

geg. $X(u,v) = \begin{pmatrix} \sin(u) \cdot \cos(v) \\ \sin(u) \cdot \sin(v) \\ \cos(u) \end{pmatrix}$

$$X_u = \begin{pmatrix} \cos(u) \cdot \cos(v) \\ \cos(u) \cdot \sin(v) \\ -\sin(u) \end{pmatrix} \Rightarrow X_\pi = \begin{pmatrix} -\cos(v) \\ -\sin(v) \\ 0 \end{pmatrix}$$

$$X_v = \begin{pmatrix} \sin(u) \cdot -\sin(v) \\ \sin(u) \cdot \cos(v) \\ 0 \end{pmatrix} \Rightarrow X_v = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$X(u,v) =$

$$g_{11} = X_\pi^2 = \begin{pmatrix} -\cos(v) \\ -\sin(v) \\ 0 \end{pmatrix}^2 = \cos(v)^2 + \sin(v)^2 = 1$$

$$g_{12} = X_\pi \cdot X_v = 0$$

$$g_{22} = X_v^2 = 0$$

$$\Rightarrow L[c] = \int_a^b |c'(t)| = \int_a^b \sqrt{\dot{c}_1(t)^2} = \int_a^b \dot{c}_1(t) = \Big[c_1(t) \Big]_a^b$$