

19.1.6415)

Beh: für $a = -c$ ist $f(x, y) = ax^2 + 2bxy + cy^2$, $(x, y) \in \mathbb{R}^2$
harmonisch in \mathbb{R}^2 .

Bew:

$f(x, y)$ ist harmonisch, wenn $\Delta f(x, y) = 0$:

$$\begin{aligned} 0 = \Delta f(x, y) &= \operatorname{div} \operatorname{grad} f(x, y) \\ &= \operatorname{div} \left(\frac{\partial f(x, y)}{\partial x}, \frac{\partial f(x, y)}{\partial y} \right) \\ &= \operatorname{div} (2ax + 2by, 2bx + 2cy) \\ &= \frac{\partial (2ax + 2by)}{\partial x} + \frac{\partial (2bx + 2cy)}{\partial y} \\ &= 2a + 2c \end{aligned}$$

$$\Leftrightarrow 2a = -2c \quad \Leftrightarrow a = -c$$