Aufgabe 8

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Integrabilitätsbedingung: $\frac{\partial f_1}{\partial x_2} = \frac{\partial f_2}{\partial x_1}$

(i) $f(x,y) = (x^2+y^2, 2xy)$ $\frac{\partial(x^2+y^2)}{\partial y} = \frac{\partial f_1}{\partial x_2} = 2y = \frac{\partial f_2}{\partial x_1} = \frac{\partial(2xy)}{\partial x}$

Potenzial: $\operatorname{grad}\varphi = f$

$$\rightsquigarrow \quad f_1(x) = x^2+y^2 = \varphi_x(x) \xrightarrow[\text{int.}]{\text{nach } x} \varphi(x) = \frac{1}{3}x^3 + xy^2 + C \quad , C \in \mathbb{R}$$

$$f_2(x) = 2xy = \varphi_y(x) \qquad \varphi_y(x) = 2xy \checkmark$$

$$\Rightarrow \varphi(x) = \frac{1}{3}x^3 + xy^2 + C \quad , C \in \mathbb{R}$$

(ii) $f(x,y,z) = (x+z, -y-z, x-y)$

$$\frac{\partial f_1}{\partial x_2} = \frac{\partial(x+z)}{\partial y} = 0 = \frac{\partial(-y-z)}{\partial x} = \frac{\partial f_2}{\partial x_1} \checkmark$$

$$\frac{\partial f_1}{\partial x_3} = \frac{\partial(x+z)}{\partial z} = 1 = \frac{\partial(x-y)}{\partial x} = \frac{\partial f_3}{\partial x_1} \checkmark$$

$$\frac{\partial f_2}{\partial x_3} = \frac{\partial(-y-z)}{\partial z} = -1 = \frac{\partial(x-y)}{\partial y} = \frac{\partial f_3}{\partial x_2}$$

Potenzial:

$$f_1(x) = x+z = \varphi_x(x) \xrightarrow[\text{int.}]{\text{nach } x} \frac{1}{2}x^2 + zx$$

$$f_2(x) = -y-z = \varphi_y(x) \xrightarrow[\text{int.}]{\text{nach } y} -\frac{1}{2}y^2 - zy$$

$$f_3(x) = x-y = \varphi_z(x) \xrightarrow[\text{int.}]{\text{nach } z} xz - yz$$

$$f_3(x) = x - y = \varphi_z(x) \xrightarrow[\text{int.}]{\text{nach } z} xz - yz$$

$$\rightsquigarrow \varphi(x) = \frac{1}{2}x^2 - \frac{1}{2}y^2 + zx - zy + C \quad , C \in \mathbb{R}$$