20.1 Aufgabe 11

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$\frac{\partial f}{\partial N} := \langle \nabla f, N \rangle$

(i) 1. Greensche Identität

Gauß'scher Divergenzsatz gilt

z.z. $\int_\Omega (f \Delta g + \langle \nabla f, \nabla g \rangle) \, dl_2(x,y) = \int_a^b f \frac{\partial g}{\partial N} |\dot{\gamma}(t)| \, dt$

Beweis:

- wende Divergenzsatz an auf $f \nabla g$

$$\int_\Omega \operatorname{div}(f \nabla g) \, dl_2(x,y) = \int_{\partial\Omega} \langle f \nabla g, N(x,y) \rangle |\dot{\gamma}(t)| dt$$

$$= \int_a^b f \langle \nabla g, N(x,y) \rangle |\dot{\gamma}(t)| \, dt$$

$$= \int_a^b f \frac{\partial g}{\partial N} |\dot{\gamma}(t)| \, dt$$

$$\int_\Omega \nabla f \nabla g + f \Delta g \, dl_2(x,y) \quad \square$$

(ii) 2. Greensche Identität

z.z. $\int_\Omega (f \Delta g - g \Delta f) \, dl_2(x,y) = \int_a^b \left(f \frac{\partial g}{\partial N} - g \frac{\partial f}{\partial N}\right) |\dot{\gamma}(t)| \, dt$

Beweis:

folgt aus der ersten Green'schen Formel

$$\int_\Omega (f \Delta g - g \Delta f) \, dl_2(x,y)$$

$$= \int_\Omega f \Delta g \, dl_2(x,y) - \int_\Omega g \Delta f \, dl_2(x,y)$$

$$\overset{(i)}{=} \int_a^b f \frac{\partial g}{\partial N} |\dot{\gamma}(t)| \, dt - \cancel{\int_\Omega \langle \nabla f, \nabla g \rangle \, dl_2(x,y)}$$

$$- \left[\int_a^b g \frac{\partial f}{\partial N} |\dot{\gamma}(t)| dt - \cancel{\int_\Omega \langle \nabla g, \nabla f \rangle \, dl_2(x,y)} \right]$$

$$= \int_a^b f \frac{\partial g}{\partial N} |\dot{\gamma}(t)| \, dt - \int_a^b g \frac{\partial f}{\partial N} |\dot{\gamma}(t)| \, dt$$

$$= \int_a^b \left(f \frac{\partial g}{\partial N} - g \frac{\partial f}{\partial N}\right) |\dot{\gamma}(t)| \, dt \quad \square$$