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Kapitel 20
Aufgabe 9

$f(x,y) = (xy, x^2 - y^2) \qquad (x,y) \in \mathbb{R}^2$

$\Omega = \{(x,y) \in \mathbb{R}^2 : 0 \leq x \leq 2, 2 \leq y \leq 5\}$

Flächenintegral:

$\operatorname{div} f(x,y) = y - 2y = -y$

$\Rightarrow \int_\Omega \operatorname{div} f(x,y) \, d\lambda_2(x,y) = -\int_0^2 \int_2^5 y \, dy\, dx = \underline{-21}$

Kurvenintegral:

$\gamma_1(t) = (t, 2), \quad t \in [0,2] \qquad \gamma_3(t) = (7-t, 5), \quad t \in [5,7]$

$\gamma_2(t) = (2, t), \quad t \in [2,5] \qquad \gamma_4(t) = (0, 12-t), \quad t \in [7,10]$

$\Rightarrow I_1 := \int_{\gamma_1} \{f_1(x,y)\,dy - f_2(x,y)\,dx\} = -\int_0^2 t^2 - 4\,dt = -\left[\frac{1}{3}t^3 - 4t\right]_0^2$

$= \frac{16}{3}$

$I_2 := \int_{\gamma_2} \{f_1(x,y)\,dy - f_2(x,y)\,dx\} = \int_2^5 2t\,dt = \left[t^2\right]_2^5 = 21$

$I_3 := \int_{\gamma_3} \{f_1(x,y)\,dy - f_2(x,y)\,dx\} = \int_5^7 24 - 14t + t^2\,dt$

$= \left[24t - 7t^2 + \frac{1}{3}t^3\right]_5^7 = -\frac{142}{3}$

$I_4 := \int_{\gamma_4} \{f_1(x,y)\,dy - f_2(x,y)\,dx\} = \int_7^{10} 0\,dt = 0$

$\Rightarrow \int_{\partial\Omega} \{f_1(x,y)\,dy - f_2(x,y)\,dx\} = I_1 + I_2 + I_3 + I_4$

$= \frac{16}{3} + 21 - \frac{142}{3} + 0 = \underline{-21}$